

Investigation on the oscillation modes in a thermoacoustic Stirling prime mover: mode stability and mode transition

Z.B. Yu ^{a,*}, Q. Li ^a, X. Chen ^b, F.Z. Guo ^b, X.J. Xie ^a, J.H. Wu ^b

^a Cryogenics Laboratory, Technical Institute of Physics and Chemistry, Chinese Academy of Science, P.O. Box 2711, Beijing, 100080 China

^b Cryogenics Laboratory, Huazhong University of Science and Technology, Wuhan 430074, China

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Abstract

The purpose of this paper is to investigate the stability of oscillation modes in a thermoacoustic Stirling prime mover, which is a combination of looped tube and resonator. Two modes, with oscillation frequencies of 76 and 528 Hz, have been observed, stabilities of which are widely different. The stability of the high frequency mode (HFM) is affected by low frequency mode (LFM) strongly. Once the LFM is excited when the HFM is present, the HFM will be gradually slaved and suppressed by the LFM. The details of the transition from HFM to LFM have been described. The two stability curves of the two modes have been measured. Mean pressure P_m is an important control parameter influencing the mode stability in the tested system.

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1. Introduction

Thermoacoustic prime mover includes three types: standing wave prime mover, loop-type traveling wave prime mover and thermoacoustic Stirling prime mover. Standing wave prime mover has attracted much attention [1–4,10]. It converts heat into acoustic power through an intrinsically irreversible thermodynamic process. Ceperley realized that a traveling acoustic wave propagating through a regenerator undergoes a thermodynamic cycle similar to Stirling cycle [5,6]. But his experimental engine was not able to amplify acoustic power. Later, Yazaki et al. first demonstrated such a prime mover, but at low efficiency [7]. Yazaki realized that this was due to the low acoustic impedance of the working gas, which caused large viscous losses. In 1999, Backhaus and Swift designed and built a Stirling-cycle based thermoacoustic prime mover. Since it employs the inherently reversible Stirling cycle, the prime mover's efficiency is much higher than that of standing wave

prime mover and loop-type traveling wave prime mover [8,9].

In Ref. [10], stability of modes in a standing wave prime mover has been studied in detail. The two modes in Ref. [10] are the fundamental and second longitudinal modes, the frequencies of which are 484.0 and 996.1 Hz, respectively. The simultaneous excitation of both modes has been observed. The mode stability of thermoacoustic Stirling prime mover has not attracted enough attention preciously. Also, two modes were observed in the thermoacoustic prime mover studied by Biwa et al. [11]. In addition they stated the transition from the standing to the traveling wave mode through quasi-periodic states by increasing the heat supplied to the prime mover. The authors realized that an increase in entropy flow determines the oscillating mode [9]. In our early work on the tested prime mover [12,13], two modes, high frequency mode (HFM, 528 Hz) and low frequency mode (LFM, 76 Hz), have been reported. A transition from HFM to LFM has been introduced briefly.

In this paper, we will concentrate on the stability of the two modes in detail. What is more, the details of the transition from HFM to LFM will be described. In addition, some new experimental results will be introduced. The characteristics of the two modes have been

* Corresponding author. Tel.: +86-10-626-25-387; fax: +86-10-625-64-049.

E-mail address: zbyu@cl.cryo.ac.cn (Z.B. Yu).

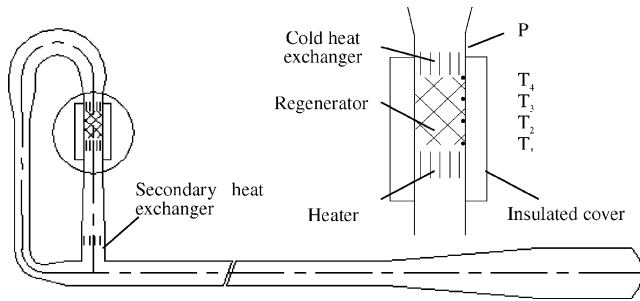


Fig. 1. Schematic presentation of the experimental apparatus.

observed and analyzed respectively. Two stability curves of them have been measured. There is a critical mean pressure P_{mcr} in the tested prime mover, where the two measured stability curves intersect. The system begins to oscillate in LFM and HFM respectively, when the mean pressure is below or above P_{mcr} .

2. Experimental apparatus

A schematic diagram of the experimental apparatus is shown in Fig. 1. The mean length of the looped tube L_1 is about 0.708 m. The length of resonator L_2 is about 1.138 m [12,13]. The instrumentation of the device is also schematically shown in Fig. 1. One piezoelectric pressure sensor is placed above the cold heat exchanger shown as P to measure the pressure wave. Four thermocouples are placed along the regenerator shown as T_1 , T_2 , T_3 and T_4 in Fig. 1.

The matrix of the regenerator is a pile of 150 mesh stainless-steel gauze. The working gas is nitrogen in the following experiments.

3. Experiments

3.1. Two modes: LFM and HFM

The input electric power of heater is kept as 95 W. The prime mover begins to oscillate in LFM and HFM respectively and reaches stationary, when the mean pressure P_m is 0.8 and 1.6 MPa. Temperatures along the regenerator and pressure amplitude versus time are shown in Figs. 2 and 3. The frequencies of the LFM and HFM mode are 76 and 528 Hz respectively. According to the discussion in Ref. [12], 76 and 528 Hz are the normal frequencies of the two subsystems, which close to the intrinsic frequencies of the resonator and the looped tube respectively.

Comparing Fig. 2 with Fig. 3, we can find the characteristics of the two modes respectively. The frequency of HFM is much higher than that of LFM, but its pressure amplitude is much smaller than that of LFM.

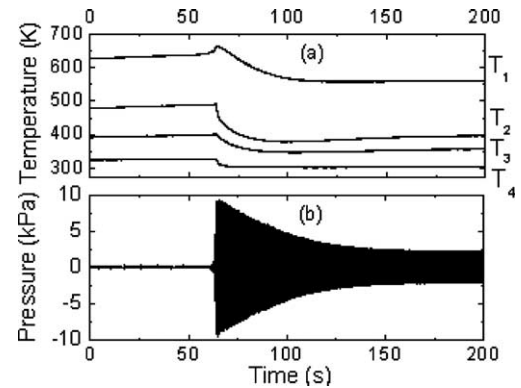


Fig. 2. 0.8 MPa, 95 W, the tested system begins to oscillate in LFM (76 Hz).

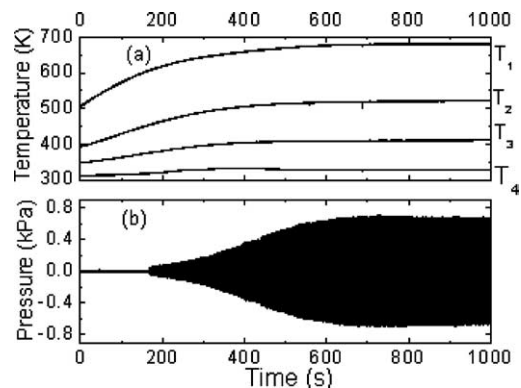


Fig. 3. 1.6 MPa, 95 W, the tested system begins to oscillate in HFM (528 Hz).

There is a sharp drop in the temperatures evolution along the length of regenerator when the engine begins to oscillate in LFM, but not in HFM.

The two experiments mentioned above revealed that two relatively stationary oscillation modes could be excited in the tested prime mover, the frequencies of which are 76 Hz and 528 Hz respectively. According to the discussion in Ref. [12], HFM is not the high order mode of LFM. What is more, the tested system begins to oscillate in different modes when the mean pressure as the control parameter has been changed with other experimental conditions been kept the same.

3.2. Mode transition: transition from HFM to LFM

All the experimental conditions and control parameters are kept the same as the experiment of Fig. 3. Only the input electric power of the heater is increased to 151 W. The experimental results are shown in Fig. 4. It is observed that the prime mover begins to oscillate in a non-stationary HFM, and then changes the oscillating mode from HFM to LFM through the quasi-periodic states involving both HFM and LFM. So does the next periods shown in Fig. 4(b). From Fig. 4(a), it can be

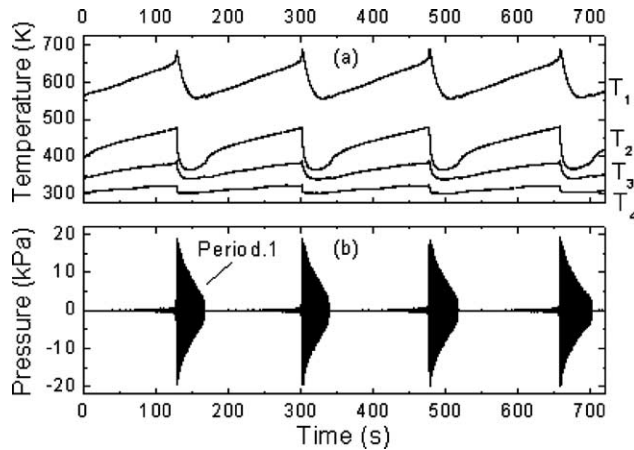


Fig. 4. 1.6 MPa, 151 W, the tested system begins to oscillate in HFM, and then changes to LFM.

found that the temperatures along the length of the regenerator also evolve periodically.

In Fig. 4, the periodic on–off oscillation has been studied in Ref. [12]. The experiment in Ref. [12] indicated that non-zero mass flow in the looped tube plays a significant role in this effect. The periodic on–off oscillation vanished, when the non-zero mass flow was suppressed partially.

The details of the transition can be found in Fig. 5. The first period of the oscillation in Fig. 4 is enlarged as (a), (b) in Fig. 5. In addition, (c) and (d) are the details after one of the component frequency in the original data have been filtered. From Fig. 5(b) and (c), it can be observed that the tested system begins to oscillate in HFM at about 65 s. The pressure amplitude rises stably in the region labeled I, which lasts about 61 s. Then the LFM is excited at about 126 s shown in Fig. 5(d). Once

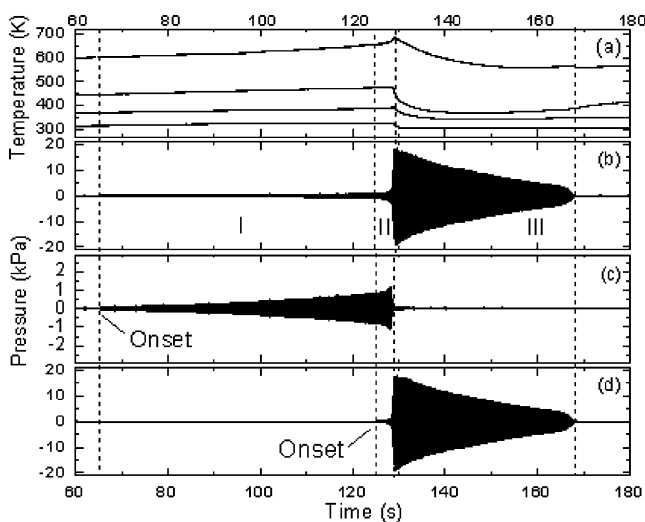


Fig. 5. The details of the transition from HFM to LFM. (a, b) The original data of the Period 1 in Fig. 4. (c) The evolution of HFM after the LFM has been filtered. (d) The evolution of LFM after the HFM has been filtered. I—HFM; II—quasi-periodic states; III—LFM.

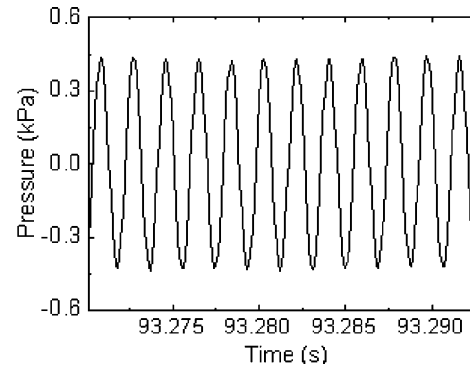


Fig. 6. The waveform of HFM shown in region I of Fig. 5.

the onset of LFM happens, the HFM starts to be gradually slaved and suppressed by LFM strongly shown in region II of Fig. 5(c) and (d). In a few seconds, the pressure amplitude of HFM attenuates to zero quickly, while the amplitude of LFM reaches the maximum after an exponential growth shown in region II of Fig. 5(b) and (d). At about 129 s the oscillating mode is changed into LFM completely shown in the region III of Fig. 5(b)–(d). Oscillation in region II is the quasi-periodic states involving both HFM and LFM, which are predominated by LFM.

From Fig. 5(a), it can be seen that the temperatures along the length of the regenerator remain increasing after the prime mover begins to oscillate in HFM. The pressure amplitude also remains stably growth synchronously, shown in Fig. 5(a) and (b). The temperatures still remain rising in region II after LFM is induced at about 126 s. The temperatures reach their top value at about 129 s together with the pressure amplitude of LFM growing up to their top value.

The waveforms in regions I and III are shown in Figs. 6 and 7, the frequencies of which are 528 and 76 Hz respectively. Fig. 8(a) and (b) shows the waveform and the Fourier components of the quasi-periodic states involving both HFM and LFM. From Fig. 8(b), two frequencies 76 Hz and 528 Hz can be seen respectively.

From Fig. 5(b)–(d), it is observed that the two stationary modes cannot coexist in the tested prime mover.

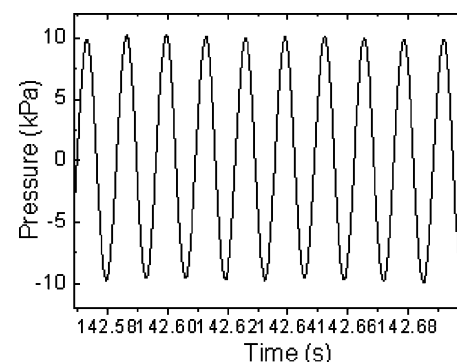


Fig. 7. The waveform of LFM shown in region III of Fig. 5.

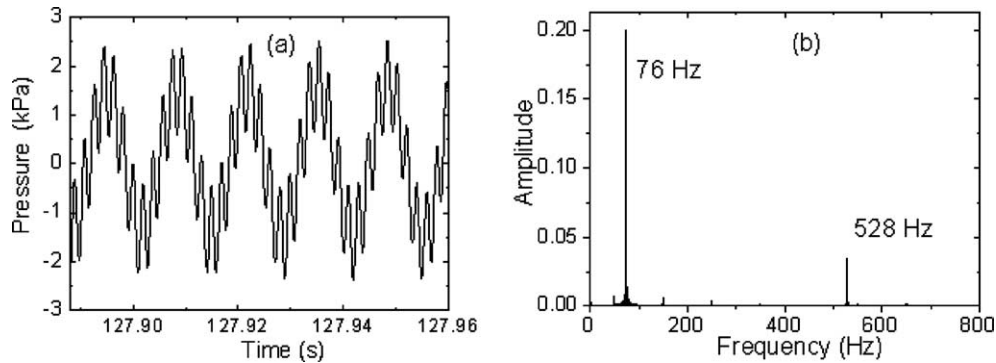


Fig. 8. (a) The waveform of quasi-periodic states shown in region II of Fig. 5, (b) the Fourier components.

Once the onset of LFM happens the oscillating mode will change from HFM to LFM quickly, since the quasi-periodic states are transient. The transition from LFM to HFM has not been observed in the tested system, although many experiments have been tried to investigate it. Further more, it has never been observed that HFM and LFM are excited simultaneously.

All the above experiment phenomena of modes are similar to those described in Ref. [10]. However, the two modes studied in Ref. [10] are the fundamental mode (480.0 Hz) and second mode (996.1 Hz) in a standing wave prime mover. Additionally, simultaneous excitation of both modes has been observed. However, in the system presented in this paper, the HFM (528 Hz) is not the second mode of the LFM (76 Hz). But they are close to the intrinsic modes of the resonator and looped tube respectively.

3.3. Mean pressure as a control parameter

The summarizing of the experimental results mentioned above reveals the significant influence of mean pressure on the onset of oscillation. In order to study this effect, we have measured the onset point of the two modes in the tested prime mover. We kept the input electric power of the heater at 151 W constantly, only changed P_m from 0.1 to 1.8 MPa with a step of 0.1 MPa. The measured plot of ΔT_{onset} versus P_m takes the form of two stability curves and is shown in Fig. 9.

In Fig. 9, the curve marked with $\circ$'s indicate the measured onset condition of LFM, while the one marked with Δ 's indicate the measured onset conditions of HFM. It can be found that the curves of LFM and HFM intersect at P_{mcr} , which divides the curves into two branches, but the left branch of HFM has not been measured because there is not a transition from LFM to HFM in the tested prime mover.

The critical mean pressure P_{mcr} is about 0.9 MPa under our experimental conditions. When $P_m \leq P_{\text{mcr}}$, the prime mover begins to oscillate in LFM. While $P_m > P_{\text{mcr}}$, two situations are observed. One (shown in

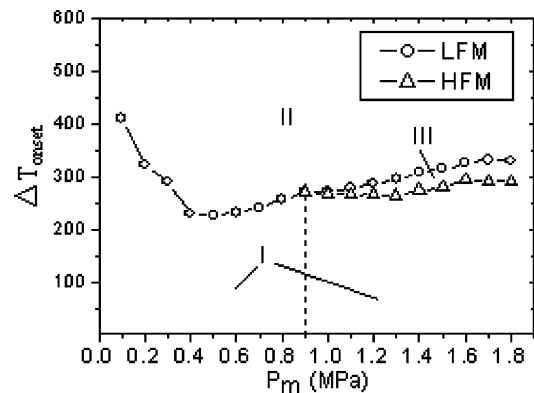


Fig. 9. Measured stability curves of HFM and LFM in the tested prime mover.

Fig. 3) is that the tested system begins to oscillating in HFM, and then reach stationary in it, when the temperature distribution of the regenerator are strictly controlled in the region III in Fig. 9. The other (shown in Fig. 4) is that the prime mover begins to oscillate in non-stationary HFM, and then changes the oscillating mode into LFM through the quasi-periodic states. So the mean pressure must be referred to as a control parameter with its threshold value at P_{mcr} .

In order for self-sustained oscillations to exist, both parameters ΔT and P_m in the tested system must lie within the region of instability, above the curves in Fig. 9. In other words, in the region labeled I, there is no oscillation in the tested prime mover. In region II, it oscillates in LFM. In region III, it oscillates in HFM, but the width of this region is very narrow. The oscillating mode will be changed from HFM to LFM through the quasi-periodic states quickly, when the temperature evolution in the regenerator is beyond region III (shown in Fig. 4).

4. Discussion

The main facts reported in this paper are the following:

- (1) Two oscillation modes can be excited, they are closely related to the intrinsic frequencies of the two subsystems—the looped tube and the resonator.
- (2) The system mean pressure plays the role of control parameter in the stability curve $\Delta T_{\text{onset}} - P_m$ of this nonlinear self-excited oscillation phenomenon, and there is a critical mean pressure where the two measured stability curves intersect, which divided the $\Delta T_{\text{onset}} - P_m$ plot into three regions, shown in Fig. 9.
- (3) The stability curve shows an asymmetry about P_{mcr} , on the left branch of which the onset mode are all LFM, while the HFM can only happen on the right branch.
- (4) Only stationary LFM can be observed under any value of the control parameter P_m (in region II Fig. 9), but the stationary HFM exist only in a strictly restrained region (in region III Fig. 9).
- (5) The temperature evolution along the regenerator differs evidently between the LFM and HFM shown in Figs. 2 and 3, this implies that the thermoacoustic process for each mode deviates also significantly.

A peculiar feature of the tested prime mover system needs to be mentioned, that the intrinsic frequencies of the looped tube is about 499 Hz (fundamental mode, $f_1 = a/L_1$), and for the resonator is about 78 Hz (fundamental mode, $f_2 = a/4L_2$), they are close to the measured frequencies of HFM (528 Hz) and LFM (76 Hz). For this case, the two subsystems with different intrinsic frequencies (partial frequencies) are coupled together to be a system with two degrees of freedom. The coupling condition plays an important role [14], but little attention has been paid on it.

The system mean pressure as a control parameter in this nonlinear self-excitation process reflects the dominant effect of relative viscous penetration depth $\sqrt{\nu/\omega}/r_h$ in the region of near-isothermal process (left branch) of stability curve. The fact, that HFM can be observed only on the right branch of the curve, implies that the value of relative viscous penetration depth identified from P_{mcr} is the boundary near-isothermal and near-adiabatic processes in this thermoacoustic system on the right branch. Beyond this boundary, inertia plays the dominant role instead of viscosity.

Among the observed HFMs in the tested system, only a part of them can be stationary (those shown in Fig. 3 and existed in region III Fig. 9), while others are non-stationary and transit to LFM as shown in Fig. 4 and Fig. 5. On the constraint condition for the stationary HFM, experimental investigation is underway and going to be published. The evident difference between the temperature evolutions along regenerator of Figs. 2 and 3 reveal the essential disparity between the LFM and HFM.

5. Conclusion

Two oscillation modes, HFM (528 Hz) and LFM (76 Hz), have been observed in the tested thermoacoustic Stirling prime mover system consisted of the looped tube and the resonator with different intrinsic frequency. The stabilities of them are widely different. The stability of HFM can be affected strongly by the LFM. Mean pressure P_m is a significant control parameter determining the threshold value of onset temperature difference. A matching between the looped tube and the resonator length ratio and the penetration depth in regenerator is needed for better performance. On the constraint condition for the stationary HFM to exist, more attention should be paid.

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